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Black Holes have Intrinsic Scalar Curvature

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The scalar curvature R is invariant under isometric symmetries (distance invariance) associated with metric spaces. Gravitational Riemannian manifolds are metric spaces. For Minkowski Space, the distance invariant is $x \cdot y$, where x, y are arbitrary 4-vectors. Thus the isometry symmetry associated with Minkowski Space is the Poincaré Group. The Standard Model Lagrangian density $L_{\rm SM}$ is also invariant under the Poincaré Group, so for Minkowski Space, the scalar curvature and the Standard Model Lagrangian density are proportional to each other. We show that this proportionality extends to general gravitational Riemannian manifolds, not just for Minkowski Space. This predicts that Black Holes have non-zero scalar curvatures $R^{\rm BH} \neq 0$. For Schwarzschild Black Holes, $R^{\rm BH}$ is predicted to be $R^{\rm BH} = -3/r_S^2$, where r_S is the Schwarzschild radius. The existence of $R^{\rm BH} \neq 0$ means that Black Holes cannot evaporate.

Keywords: Joint action; scalar curvature.

1. Issues with the Einstein–Hilbert Joint Action

All experiments are consistent with Einstein's choice of the gravitational Lagrangian density L_G

$$L_G = R/2\kappa, \tag{1}$$

where we use the $(-+++)$ metric signature and the constant $\kappa = \frac{8\pi G}{c^4}$, where R is the scalar curvature, G is Newton's gravitational constant and c is the speed of light. Every available gravity experiment has confirmed that this is the correct gravitational Lagrangian density. However, General Relativity requires a second, equally important and independent physics choice: What is the correct joint gravity–matter action $A_{\rm joint}$? For this problem, Einstein chose the linear Einstein–Hilbert joint action $A_{\rm EH}$

$$A_{\rm joint} \stackrel{?}{=} A_{\rm EH} = \int \left\{ \frac{R}{2\kappa} + L_{\rm SM} \right\} \sqrt{-g}\, dx^3 dt \tag{2}$$

(Of course, Einstein does not have the question mark ? above the equal sign). In this equation, $g = \det[g_{\mu\nu}]$ and L_{SM} is the Standard Model Lagrangian density. A_{EH} is called the Einstein–Hilbert action. It turns out that this choice for A_{joint} has issues which we discuss below (All String Theory uses the Einstein–Hilbert joint action). The outstanding issue is that A_{EH} does not vanish as $L_{\mathrm{SM}} \to 0$. When there is no matter/energy, there is no gravity and no space–time, so there exists a *boundary condition*: A_{joint} must vanish as $L_{\mathrm{SM}} \to 0$. For A_{EH}, however, with vanishing L_{SM}, A_{EH} still has gravitational Riemannian manifolds because it is a linear relationship between the two Lagrangian densities.

There exist otherA_{EH} issues:

(1) A_{joint} is the link gravity has with Quantum Mechanics and the Einstein–Hilbert joint action A_{EH} is non-renormalizable in four space–time dimensions.[1,2] Non-renormalizability means that at some point, Unitarity is broken.

(2) The action is the phase of the complex quantum mechanical probability amplitude. The linear relationship between the gravitational action and the matter action in A_{EH} means that the gravitational phase and the matter phase are added together. The sum of phases means that the two complex probability amplitudes are multiplied together. This leads to

$$|\phi_{\mathrm{EH}}\rangle_{\mathrm{pure}} = |\phi_G\rangle|\phi_{\mathrm{SM}}\rangle, \tag{3}$$

where $|\phi_{\mathrm{EH}}\rangle_{\mathrm{pure}}$ is a pure vector in the Hilbert Space $\mathcal{H}_G \otimes \mathcal{H}_{\mathrm{SM}}$, with $\mathcal{H}_G$ the Hilbert Space associated with the Einstein gravitational action and $\mathcal{H}_{\mathrm{SM}}$ the complicated Standard Model Hilbert Space. But $|\phi_{\mathrm{EH}}\rangle_{\mathrm{pure}}$ is the *joint* probability amplitude for gravitational events and mass events to simultaneously occur together, and the product of wavefunctions means that these events are *uncorrelated* for pure states. No one believes that for pure states, gravity and matter are uncorrelated, which gives the wrong probability statistics.

We now explore nonlinear joint actions $A_{\mathrm{nonlinear}}$.

2. Nonlinear Joint Action

In Ref. 3, a non-polynomial (NP) nonlinear action was investigated for the joint gravity–matter action.

$$A_{\mathrm{NP}} = \int e^{\frac{R}{2QL_{\mathrm{SM}}}} L_{\mathrm{SM}} \sqrt{-g}\, d^3 dt, \tag{4}$$

where $L_{\mathrm{SM}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ is the Standard Model. Q is a constant to be determined. Equation (4) satisfies $A_{\mathrm{NP}} \to 0$ when $L_{\mathrm{SM}} \to 0$ because $L_{\mathrm{SM}} \overset{\text{static limit}}{=} -\zeta_{\mathrm{SM}}$ where ζ_{SM} is the energy density.

The variation $\delta g^{\mu\nu}$ of the non-polynomial joint gravity–matter action is satisfied by two contributions to the scalar curvature R: $g^{\mu\nu}R^{\text{field}}_{\mu\nu} = N^{\text{field}}$, where $R^{\text{field}}_{\mu\nu}$ is the Ricci curvature tensor of space–time and $N^{\text{intrinsic}}$, which have to satisfy the following equations for the variation $\delta g^{\mu\nu}$ to always be zero:

$$R^{\text{intrinsic}} = \frac{8\pi G}{c^4} L_{\text{SM}}, \tag{5}$$

$$R^{\text{field}}_{\mu\nu} - \frac{R^{\text{field}}}{2} g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{6}$$

with $Q = 12\frac{\pi G}{c^4}$ and $T_{\mu\nu}$ the stress–energy tensor. Ordinarily, the constant Q would be expected to be the constant $\kappa = \frac{8\pi G}{c^4}$ appearing in Eq. (1), but both the intrinsic scalar curvature $R^{\text{intrinsic}}$ and the field scalar curvature R^{field} contribute to Newton's gravitational law.[3] In Quantum Mechanics, Eq. (5) becomes an operator equation and the renormalization of the scalar curvature $\langle \text{groundstate}|R^{\text{intrinsic}}|\text{groundstate}\rangle$ was given by the theorem in Ref. 3.

We now want to explain Eq. (5) in more detail. In metric spaces, the scalar curvature R is invariant under the set of symmetries that leave invariant the distance measure. For Minkowski Space, the distance invariant is $x \cdot y \equiv -x_1 y_1 + \sum_{i=2}^{4} x_i y_i$ where x, y are arbitrary 4-vectors. Thus the isometry symmetry associated with Minkowski Space is the Poincaré Group. The Standard Model Lagrangian density L_{SM} is also invariant under the Poincaré Group, so for Minkowski Space, the scalar curvature and the Standard Model Lagrangian density are proportional to each other. Equation (5) states that $R^{\text{intrinsic}}$ and L_{SM} are proportional to each other for general gravitational Riemannian manifolds and not just for Minkowski Space. The constant of proportionality $\frac{8\pi G}{c^4}$ involves G, Newton's constant.

3. The Difference Between $R^{\text{intrinsic}}$ and R^{field}

General Relativity has a contradiction when applied to electromagnetism (EM), where now $T_{\mu\nu} \to T^{\text{EM}}_{\mu\nu}$ with $T^{\text{EM}}_{\mu\nu}$ the electromagnetic stress–energy tensor. It happens when we take the trace of Eq. (6): Taking the trace of this equation and using the fact that T^{EM} is traceless, we derive

$$R^{\text{field}}_{\text{EM}} = 0. \tag{7}$$

This is not the correct R for the following reason. The zero trace of $T^{\text{EM}}_{\mu\nu}$ is due to conformal symmetry, but electromagnetism is not conformally invariant. Under the scalar transformation λ_c of the conformal symmetry $\mathbf{x} \to \lambda_c \mathbf{x}$, energies En transform as $\text{En} \to \text{En}/\lambda_c$ and the only invariant energies are $\text{En} = 0$ or $\text{En} = \infty$. Finite energy plane waves have finite wave-trains caused by the electromagnetic currents J_μ beginning transmission and ending transmission. The required J_μ for finite energies break conformal symmetry. We now show how the existence of $R^{\text{intrinsic}}$ resolves this

Table 1. Some representative values of the intrinsic scalar curvature $R^{\text{intrinsic}}$.

EM field value	$R^{\text{intrinsic}}$ (in m^{-2})
Magnetic Induction Field 1 Tesla	-8.2627×10^{-38}
Electric field 2000 V/m	3.6774×10^{-48}

conundrum. For electromagnetism (in Gaussian units)

$$L_{\text{SM}} \rightarrow L_{\text{SM}}^{\text{EM}} = \frac{1}{8\pi}(\mathbf{E}^2 - \mathbf{B}^2). \tag{8}$$

We use Eq. (5) to predict $R_{\text{EM}}^{\text{intrinsic}}$ for some laboratory EM fields in MKS units in Table 1. We see that a magnetic field R has a critical minus sign difference from an electric field R. The existence of $R^{\text{intrinsic}}$ removes the incorrect value of R given by $R^{\text{field}} = 0$.

4. Black Hole Intrinsic Scalar Curvature

The vacuum equations $R_{\mu\nu}^{\text{field}} = 0$ for space–time regions do not restrict Black Holes having intrinsic scalar curvatures, since this field equation is outside the Black Hole. We already said that $R^{\text{intrinsic}}$ contributed to Newton's gravity law by changing the constant Q.

We now predict that Black Holes have intrinsic scalar curvatures by calculating R^{BH} for the Schwarzschild solution: radius $= r_S = 2\,\text{GM}/\text{c}^2$. Using Eq. (5)

$$\begin{aligned} R^{\text{BH}} &= \langle \text{BH}|R^{\text{intrinsic}}|\text{BH}\rangle \\ &= \langle \text{BH}|\frac{8\pi G}{c^4} L_{\text{SM}}|\text{BH}\rangle \\ &= -\frac{8\pi G}{c^4}\frac{M_{\text{BH}}c^2}{\frac{4}{3}\pi r_S^3} \\ &= -3/r_S^2. \end{aligned} \tag{9}$$

The implication of $R^{\text{BH}} \neq 0$ will now be discussed.

4.1. *An intrinsic Black Hole scalar curvature prevents Black Hole evaporation*

If a Black Hole were to evaporate, it would have to reach the vacuum solution $\langle 0|R^{\text{intrinsic}}|0\rangle = \langle 0|R^{\text{field}}|0\rangle = 0$, meaning *nothing remains*. However, as the *mass of the Black Hole becomes smaller*, Eq. (9) shows that

$$M^{\text{BH}} \rightarrow 0, \quad R^{\text{BH}} \rightarrow -\infty. \tag{10}$$

A $R^{\text{BH}} \neq 0$ can never reach the vacuum state; the existence of $R^{\text{BH}} \neq 0$ prevents the Black Hole from evaporating. This gives rise to a new set of scalar curvature mechanics, which will be discussed in Table 2.

4.2. *Hawking and unruh description*

Unruh[4] showed that an accelerated observer in Minkowski space sees a vacuum thermal bath which has a kinematic temperature proportional to the acceleration. However, the Unruh kinematic temperature is not dynamical: one cannot extract finite energy from the vacuum, leaving the vacuum in a lower energy state. Hawking[5] showed that the Schwarzschild metric, under Wick rotation $t \to it$, gives rise to a metric of $\mathcal{R}^2$ in angular coordinates leading to a kinematic Black Hole temperature inversely proportional to its mass. If this were a true temperature, it would give rise to a negative heat capacity[6,7]: an added δMc^2 energy lowers the temperature. Both the Hawking and Unruh effects should be considered kinematic temperatures and not temperatures associated with statistical degrees of freedom.

5. Black Hole Scalar Mechanics

The existence of $R^{\mathrm{BH}} \neq 0$ changes the Black Hole Thermodynamics into Black Hole Scalar Curvature Dynamics. The description of Black Hole Thermodynamics is given in Ref. 8. The kinematic Hawking temperature gets replaced by the Black Hole scalar curvature.

6. Conclusion

The joint gravitation and matter functional A_{joint} is the physical quantity that relates gravity and quantum mechanics. The renormalization of the scalar curvature was given in Ref. 3 using a non-polynomial A_{joint}. It was not appreciated in the literature that A_{joint} must vanish for vanishing Standard Model Lagrangian density L_{SM}. This constraint requires that the gravitational Lagrangian density L_G and L_{SM} must be in a nonlinear relationship. In contradistinction, the widely used Einstein–Hilbert joint action is linear in L_G and L_{SM}. The non-polynomial joint action in Eq. (4) predicts that Black Holes have intrinsic scalar curvatures, which is not precluded by the Einstein field equations. It predicts that electromagnetic fields have non-zero scalar curvatures, in contrast to $R_{\mathrm{EM}}^{\mathrm{field}} = 0$, which is a historical conundrum.

Table 2. Black Hole scalar curvature mechanics. M is the Black Hole mass, ξ is the surface gravity, A is the horizon area, Ω is the angular velocity, J is the angular momentum, Φ is the electrostatic potential, $\mathcal{Q}$ is the electric charge and R^{BH} is the Black Hole scalar curvature.

Law	Black hole mechanics
0	In equilibrium, R^{BH} is a constant
1	$\delta M = \frac{\xi}{8\pi}\delta A + \Omega \delta J + \Phi \delta \mathcal{Q}$
2	$\delta A \geq 0$
3	$R^{\mathrm{BH}} \not\to 0$ (in finite number of steps)

Furthermore, $R^{\mathrm{BH}} \neq 0$ prevents Black Holes from evaporating, removing the non-Unitarity debate.[9]

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